

①

## Dc machines

### I. Theoretical questions

Q1 : Draw the construction of a Dc machine showing its components.

Q2 : What is the role of Commutator & brushes in Dc machines.

Q3 : Derive the emf equation ( $E_a$ ) in Dc machines

Q4 : Show the relation between  $\Theta_e$  &  $\Theta_p$ , then, Derive an expression relating the frequency ( $f_e$ ) to the mechanical speed( $n$ )

Q5 : Show flux distribution in Dc machines.

Q6 : Describe the operation of Dc machine as a generator & as a motor

Q7 : Draw the circuit diagram of (2)  
Dc generators, then show why  
the external c/c's of shunt Dc  
generator is more drooping than  
separately excited.

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Q8 : Draw the circuit diagram of  
various types of Dc motors.

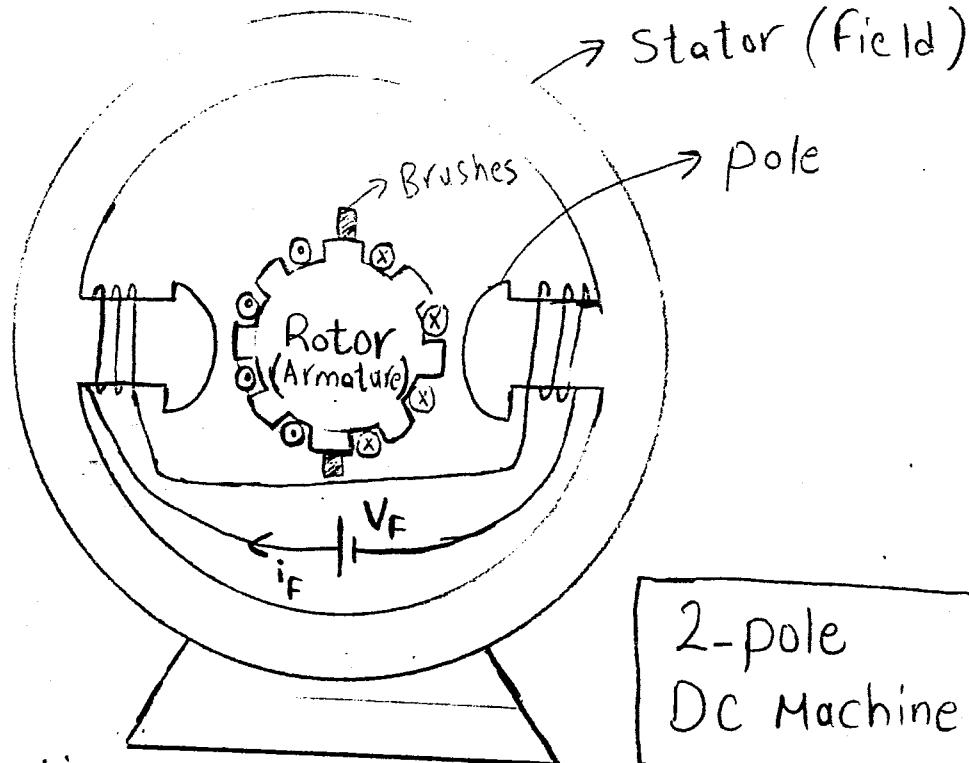
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Q9 : What is the significance (role) of  
back emf in Dc machines.

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Q10 : Why is the starting current  
is high in Dc machines, How  
to limit this current.

## Dc Machines



2-pole  
DC Machine

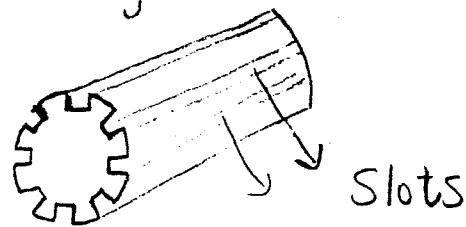
### Construction

#### ① Stator

- Carries the Field Winding (exciter)
- Field Winding is Connected to Dc Voltage Source
- Field Winding produces the magnetic Flux.

## (2) Rotor

- Carries armature Winding
- Rotor has a cylindrical shape with slots
- The conductors are placed in these slots.
- e.m.f is induced on the terminals of armature winding.



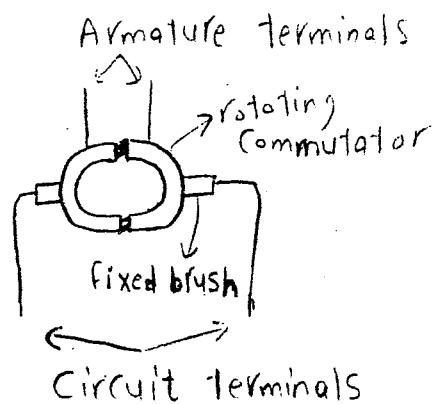
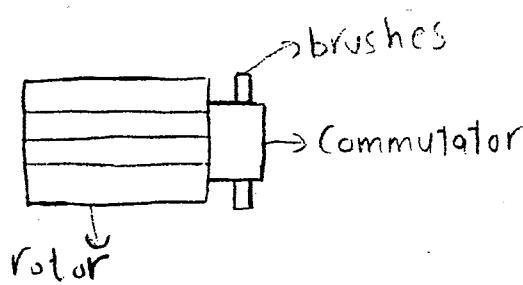
## (3) Commutator

- \* It's a Copper cylinder divided into isolated segments
- \* It's connected to armature winding terminals
- \* It's a rotating part.

(5)

#### (4) Carbon brushes

- They are fixed contacts
- They are in direct contact with the Commutator



#### (5) Air gap:

- It's the clearance between Stator and Rotor

Answer of Q2

#### Notes

(a) Commutator is used to

- Convert AC Voltage to DC Voltage in case of DC generator
- produce unidirectional torque in DC motor

(b) The horizontal axis is called pole axis,  
The Vertical axis is called interpole axis.

Answer of Q3

(6)

The Flux linkage of armature conductor:

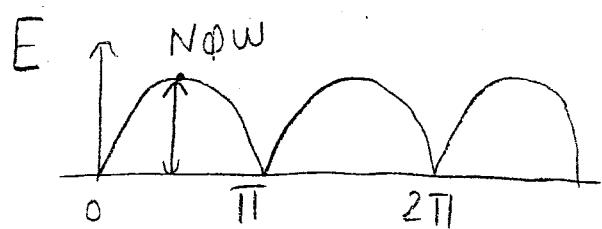
$$\lambda = N\phi \cos \omega t$$

$$\text{but } e_i = -\frac{d\lambda}{dt} = +N\phi \omega \sin \omega t$$

\* The Commutator produces DC Voltage

$$\therefore E_a = \frac{2 N \phi \omega}{\pi}$$

average  
armature  
Voltage



$$\therefore E_a = K n \phi$$

$$E_a \propto n \phi$$

$$\text{But } \phi \propto I_f$$

$$\therefore E_a \propto n I_f$$

$$\text{IF } I_f = 0 \Rightarrow E_a = 0$$

$$\text{IF } n = 0 \Rightarrow E_a = 0$$

# Answer of Q4

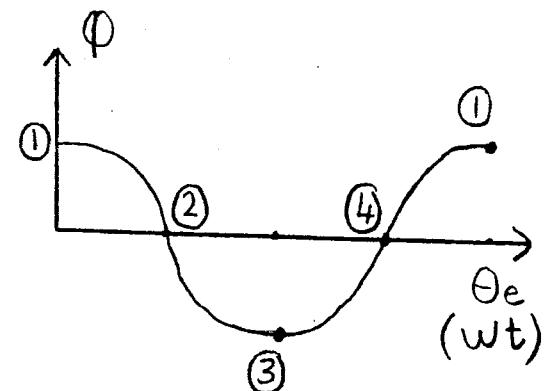
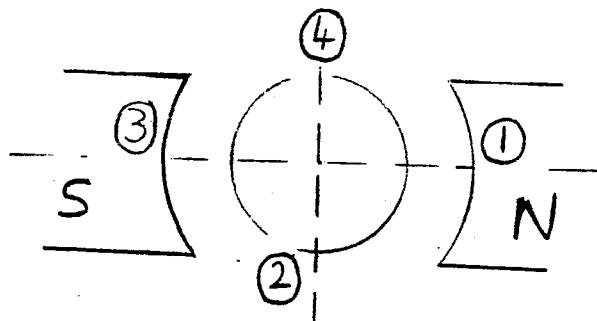
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## Relation between $\Theta_e$ , $\Theta_m$

$\Theta_e = \omega t$ ;  $\omega$  is electrical angular frequency

$\Theta_m = \omega_m t$ ;  $\omega_m$  is mechanical angular frequency

For 2 poles ( $P=1$ ) no. of pole pair



$$\underline{1 \rightarrow 2} \quad \Theta_m = 90^\circ \quad \Theta_e = 90^\circ$$

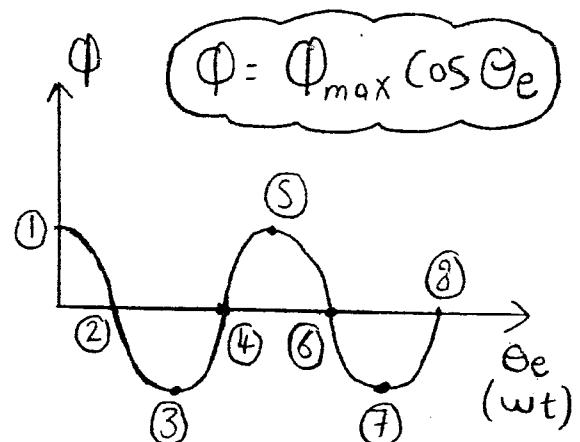
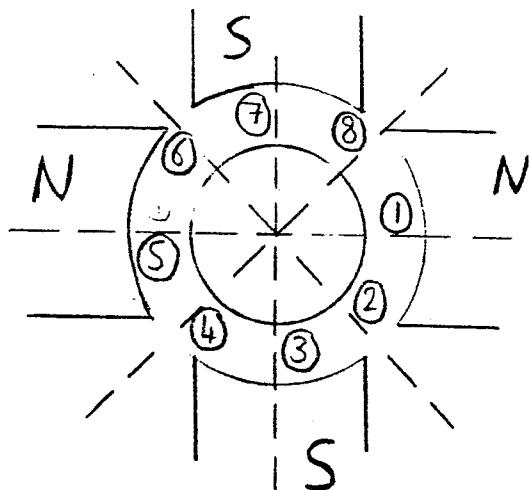
$$\underline{2 \rightarrow 3} \quad \Theta_m = 90^\circ \quad \Theta_e = 90^\circ$$

$\therefore \Theta_m = \Theta_e$

For 2 poles

(8)

For 4 poles ( $P = 2$ )



$$① \rightarrow ② \quad \Theta_m = 4S \quad \text{but} \quad \Theta_e = 90^\circ$$

$$② \rightarrow ③ \quad \Theta_m = 4S \quad \text{but} \quad \Theta_e = 90^\circ$$

$$\therefore \Theta_e = 2\Theta_m$$

$\therefore$  For ( $P$ ) poles

$$\Theta_e = -P \times \Theta_m$$

↔ bias

$$\therefore \omega_e = -P \omega_m$$

↔

- When  $P=1 \Rightarrow \Theta_e = \Theta_m, \omega_e = \omega_m$
- When  $P=2 \Rightarrow \Theta_e = 2\Theta_m, \omega_e = 2\omega_m$

## Relation between $F_e$ & $n_m$

(9)

$$\therefore \Theta_e = p\Theta_m$$

$$\therefore \frac{d\Theta_e}{dt} = p \frac{d\Theta_m}{dt}$$

$$\omega_e = p\omega_m$$

$$\therefore 2\pi F_e = p * 2\pi f_m$$

$$\therefore F_e = p f_m$$

$$\text{But } f_m = \frac{n}{60}$$

Where n: Number of revolutions per minute (rpm)

$$\therefore F_e = \frac{pn}{60}$$

$F_e$ : electrical frequency (Hz)

p: Number of pole pairs

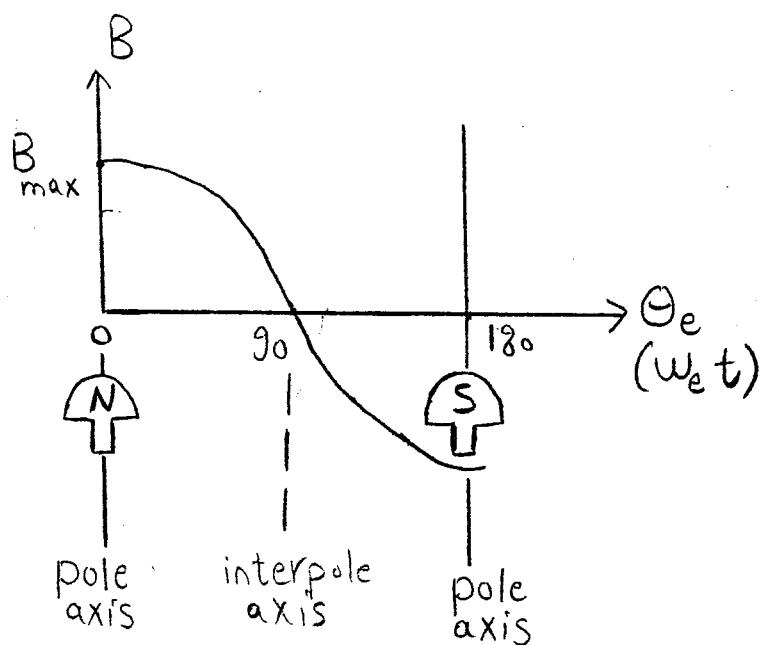
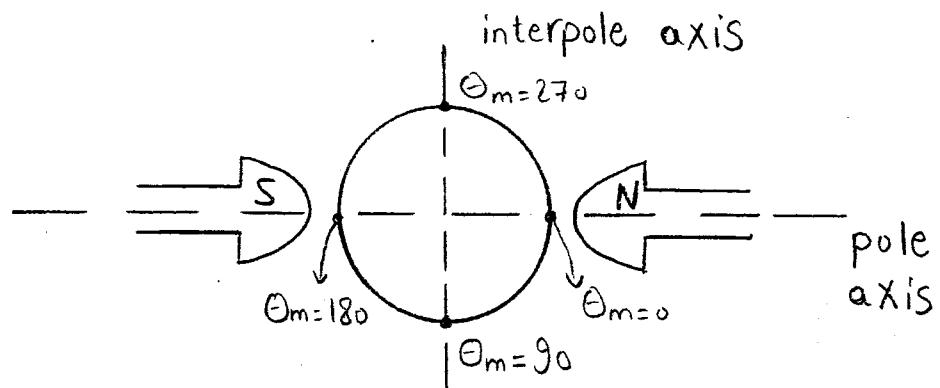
n: Mechanical speed of rotor in rpm

(5')

Answer of Q5

10

Flux density distribution in DC machine  
[Flux of stator]

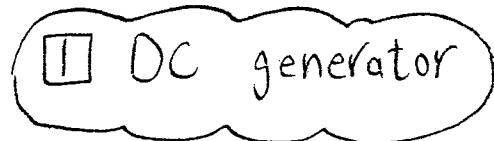


$$\therefore B = B_{max} \cos(\omega_e t)$$

This means that the Flux varies sinusoidal

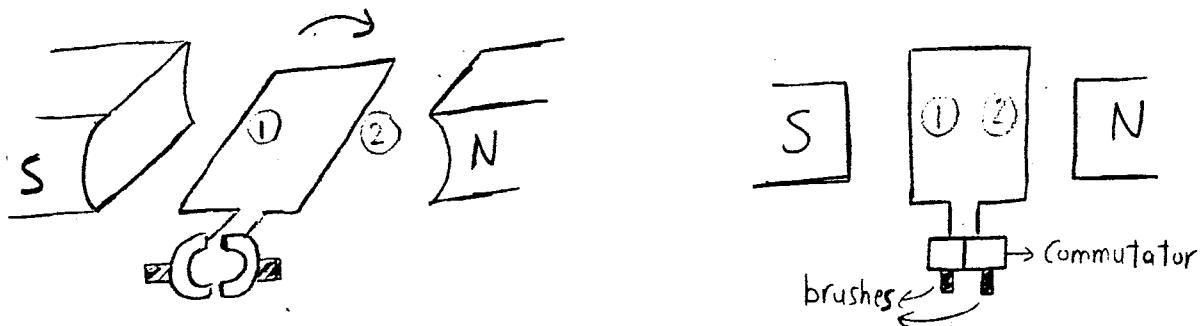
DC machine can operate as

- ① DC generator
- ② DC motor



Theory of operation

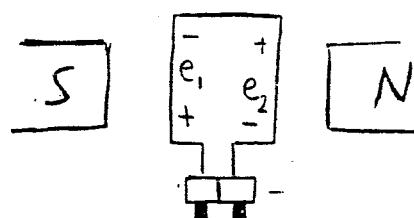
- a) The field current produces flux
- b) If the rotor (armature coil) is externally rotated



- c) A Voltage will be induced on both conductors (1,2)

$$e_1 = BLV \quad e_2 = BLV$$

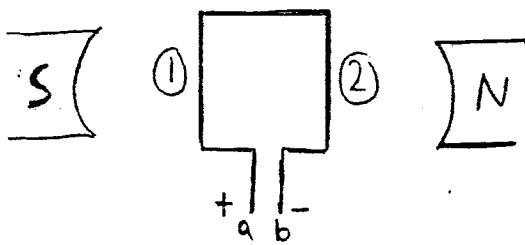
$$e_{\text{coil}} = 2BLV$$



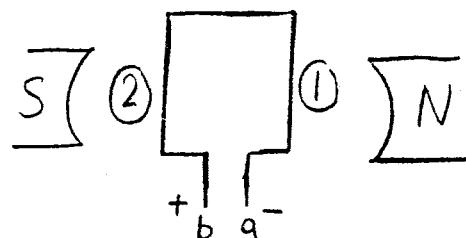
- d) The Commutator Converts Ac Voltage into Dc.

## Why do we use Commutator & brushes

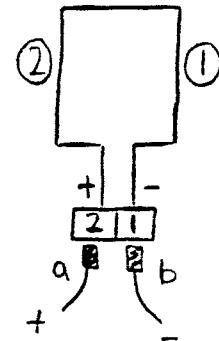
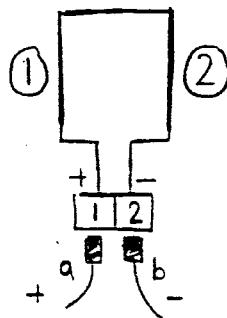
In First half cycle  
of the coil rotation



In Second half cycle  
of the coil rotation



The Voltage across the coil changes its polarity, So we use Commutator and brushes as a mechanical rectification to get DC Voltage from the coil

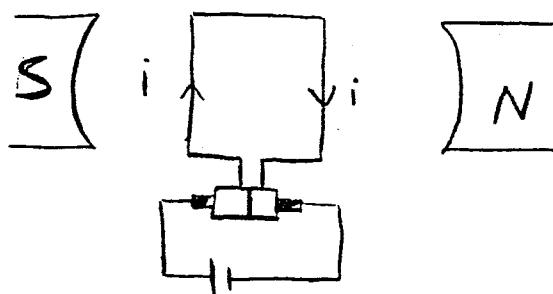


- \* brush ① is +Ve in both half cycles
- brush ② is -Ve " "

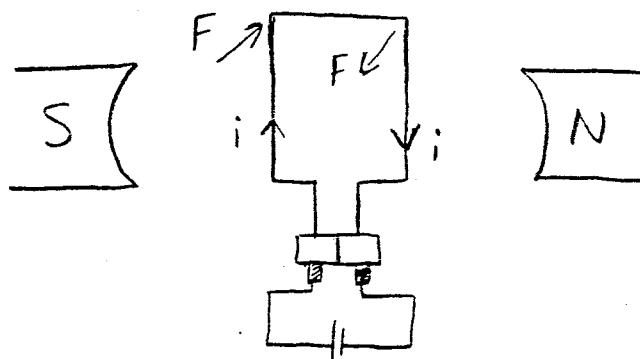
## ② DC motor

### Theory of operation

- (a) The field current produces Flux
- (b) If the armature winding terminals are connected to an external DC source, then a current will flow in armature winding.



- (c) Now both conductors (1,2) has a current  $i$  and placed in a magnetic field ( $B$ ), so a force will be produced on both conductors but in opposite direction. ( $F = BIL$ )

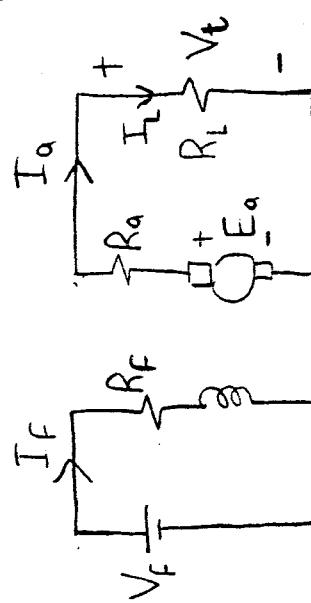


- (d) So, the coil starts to rotate with a torque  $T = BILW$ .
- (e) As the coil rotates in a magnetic field, so a back emf will be induced on the coil (as in generator).
- (f) The commutator converts the torque into unidirectional torque.

Answer of Q7

Types of DC generators

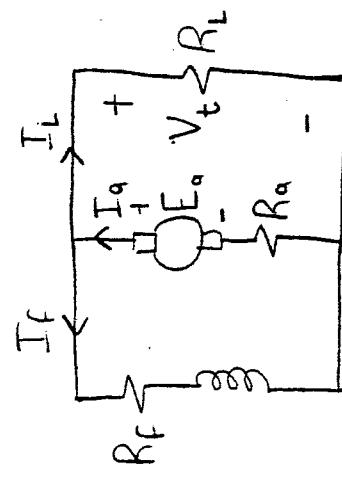
**I** Separately excited



$$E_a = V_t + I_a R_q$$

$$V_f = I_f R_f$$

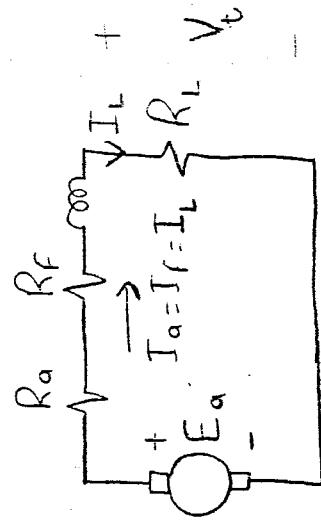
**II** Shunt excited



$$E_a = V_t + I_a R_q$$

$$I_f = \frac{V_t}{R_f}$$

**III** Series



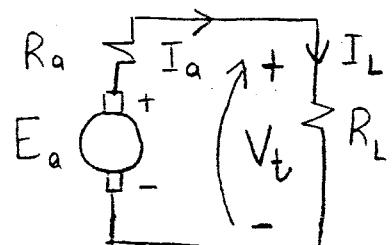
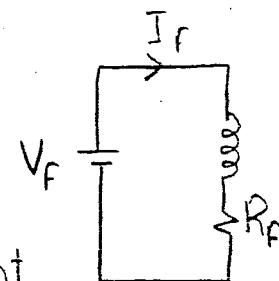
$$E_a = V_t + I_a (R_a + R_f)$$

$$I_a = I_L = I_f$$

## I Separately excited DC generator

$\Rightarrow$  Field Circuit

$$V_F = I_F R_F$$



\*  $I_F \rightarrow$  Field Current

\*  $R_F \rightarrow$  Field resistance

$\Rightarrow$  Armature Circuit

$$E_a = V_t + I_a R_a + (\text{Armature reaction drop})$$

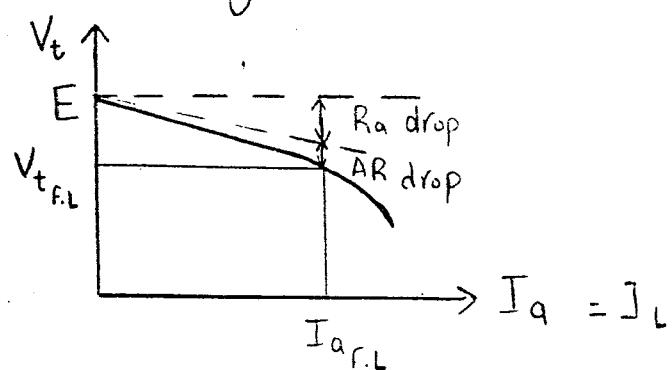
\*  $R_a \rightarrow$  Armature resistance

\*  $I_a \rightarrow$  Armature Current

\*  $V_t \rightarrow$  Terminal Voltage

If (AR) is neglected  $\Rightarrow E_a = V_t + I_a R_a$

$\Rightarrow$  External characteristic of Separately excited DC generator

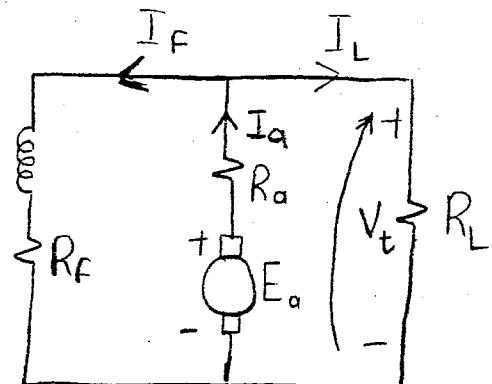


II

Shunt DC generator

$$I_F = \frac{V_t}{R_F}$$

$$E_a = V_t + I_a R_a$$



The Voltage drop is due to

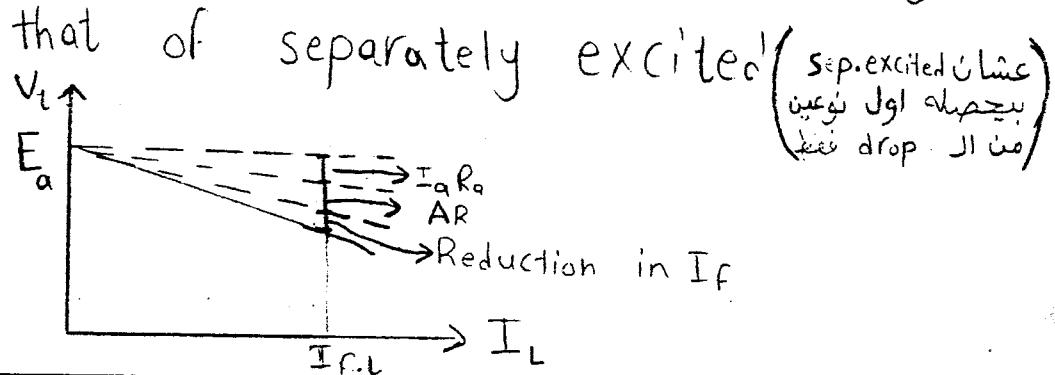
- ①  $R_a$  drop.
- ② Armature reaction drop
- ③ Decrease in field current

as  $I_F = \frac{V_t}{R_F}$  ( $V_t$  decrease as  $I_L$  increases)

$$\therefore I_F \downarrow \rightarrow \Phi \downarrow \rightarrow (E_a = K_n \Phi) \downarrow \rightarrow V_t \downarrow$$

That's why Shunt DC generator has external C/C's which is more drooping than that of separately excited

External  
C/C's

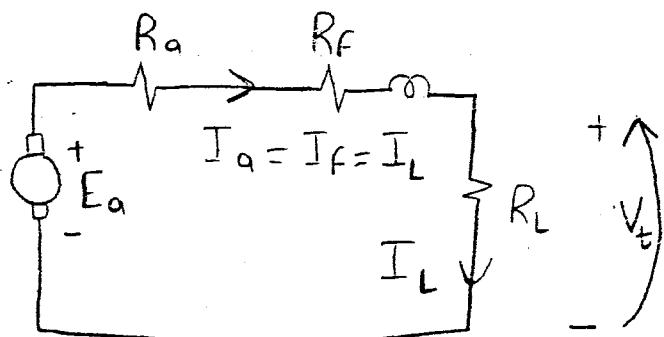


III

### Series DC generator

$$I_a = I_f = I_L$$

$$E_a = V_t + I_a(R_a + R_f)$$

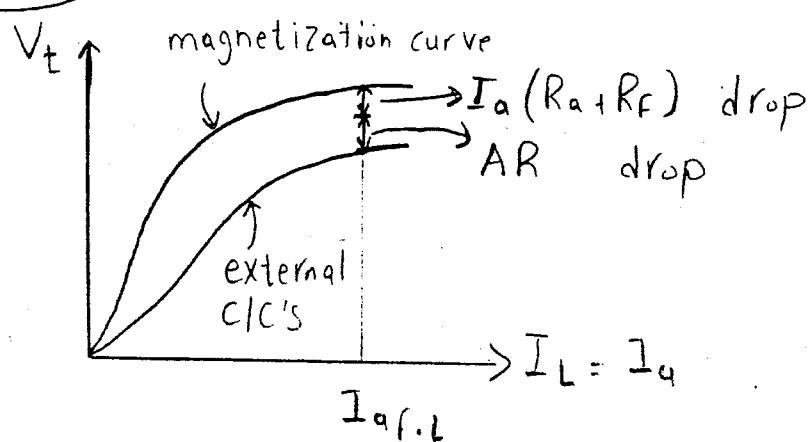


The Voltage drop is due to

$$\textcircled{1} (I_a R_a + I_a R_f) \text{ drop}$$

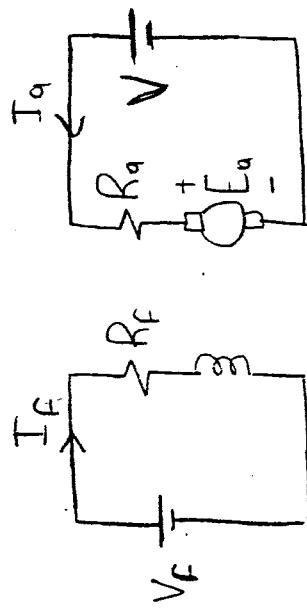
$$\textcircled{2} \text{ AR drop}$$

### External C/C's



## Types of DC motors

**[I] Separately excited**



$$V_F = T_q \frac{1}{T}$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

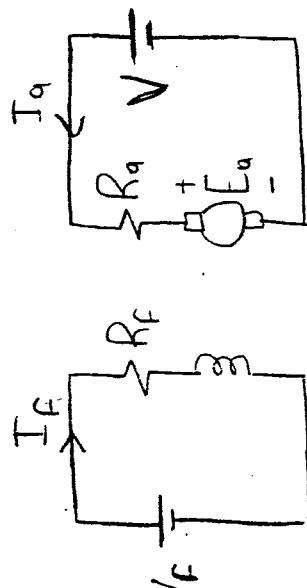
$$E_a = K_n \phi \Rightarrow E_a \propto n I_F$$

$$T = K T_a \phi \Rightarrow T \propto T_a I_F$$

$$T = K T_a \phi \Rightarrow T \propto T_a I_F$$

$$T = K T_a \phi \Rightarrow T \propto T_a I_F$$

**[II] Shunt**



$$V_F = T_q \frac{1}{T}$$

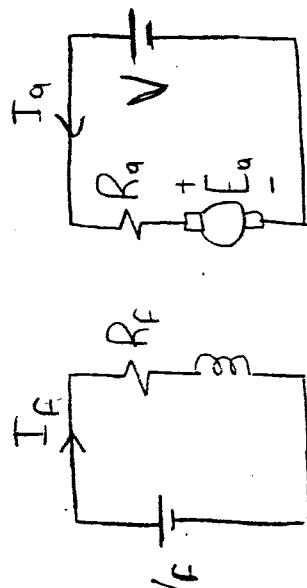
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[III] Series**



$$V_F = T_q \frac{1}{T}$$

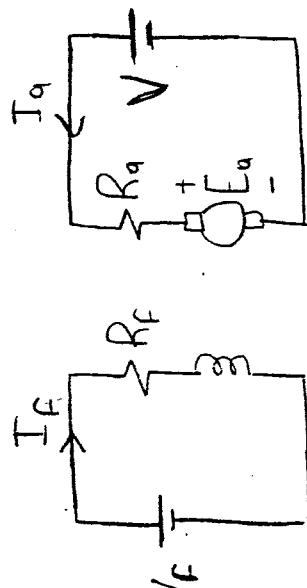
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[IV] Series**



$$V_F = T_q \frac{1}{T}$$

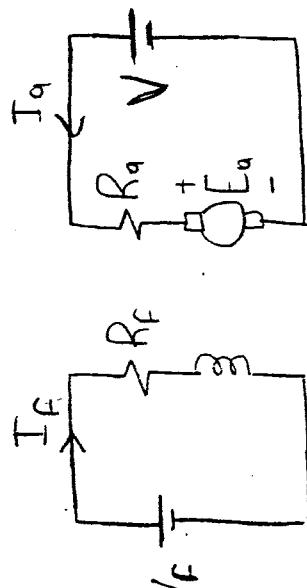
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[V] Series**



$$V_F = T_q \frac{1}{T}$$

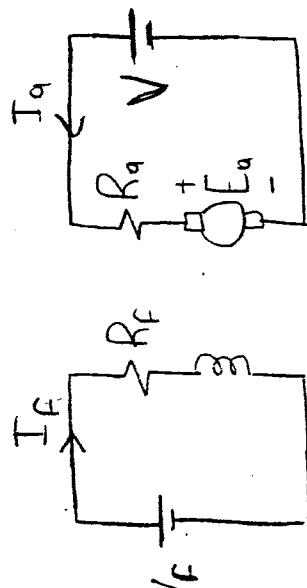
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[VI] Series**



$$V_F = T_q \frac{1}{T}$$

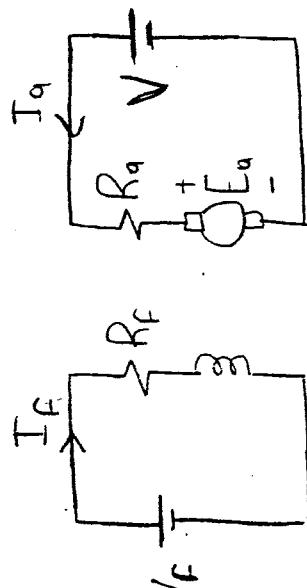
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[VII] Series**



$$V_F = T_q \frac{1}{T}$$

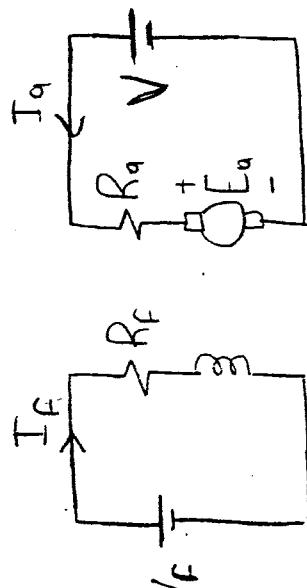
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[VIII] Series**



$$V_F = T_q \frac{1}{T}$$

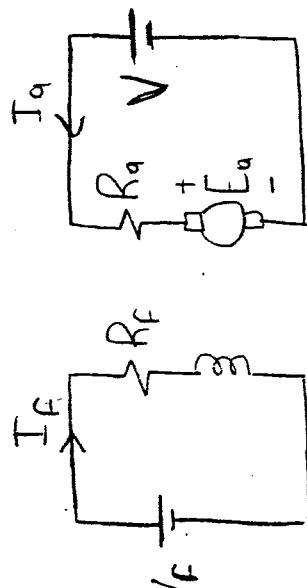
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[IX] Series**



$$V_F = T_q \frac{1}{T}$$

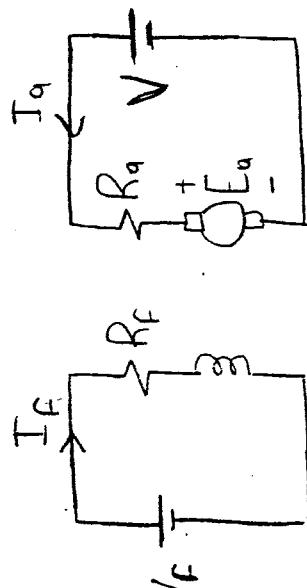
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[X] Series**



$$V_F = T_q \frac{1}{T}$$

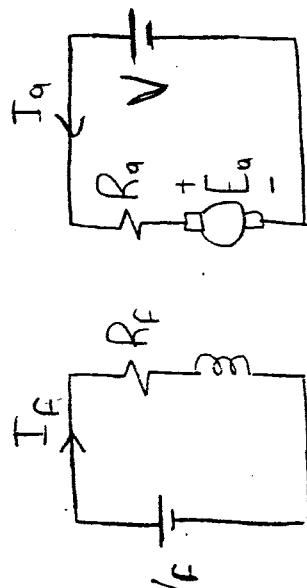
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[XI] Series**



$$V_F = T_q \frac{1}{T}$$

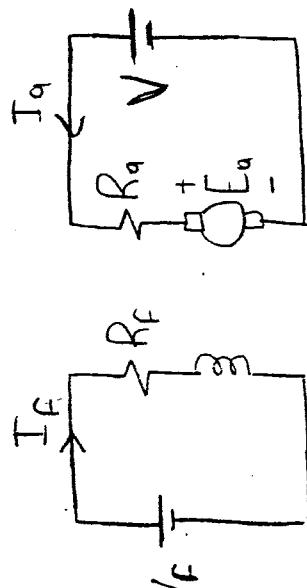
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[XII] Series**



$$V_F = T_q \frac{1}{T}$$

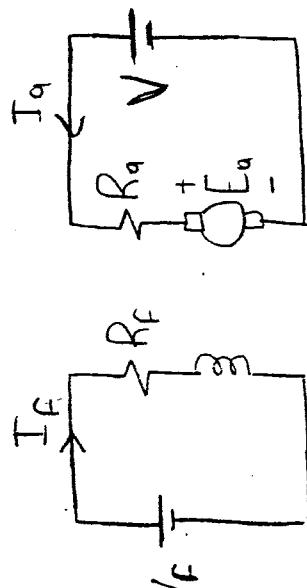
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[XIII] Series**



$$V_F = T_q \frac{1}{T}$$

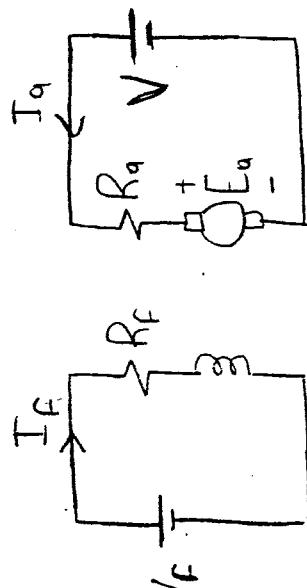
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[XIV] Series**



$$V_F = T_q \frac{1}{T}$$

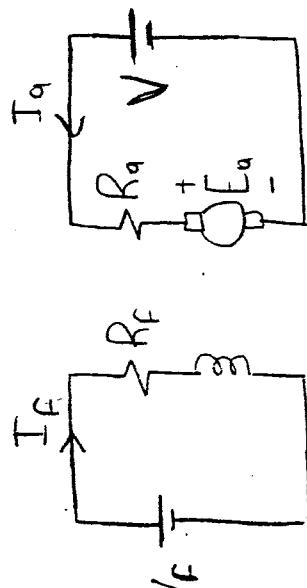
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[XV] Series**



$$V_F = T_q \frac{1}{T}$$

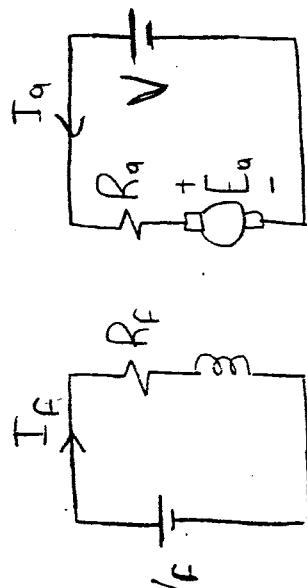
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[XVI] Series**



$$V_F = T_q \frac{1}{T}$$

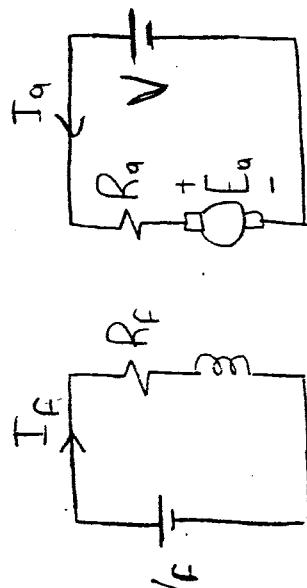
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[XVII] Series**



$$V_F = T_q \frac{1}{T}$$

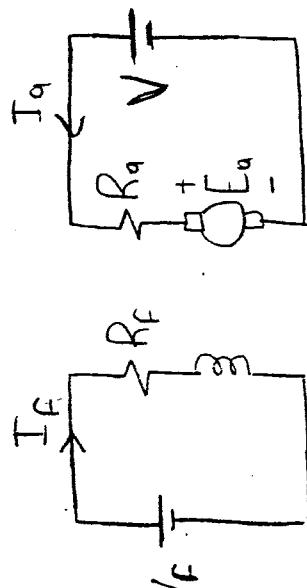
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[XVIII] Series**



$$V_F = T_q \frac{1}{T}$$

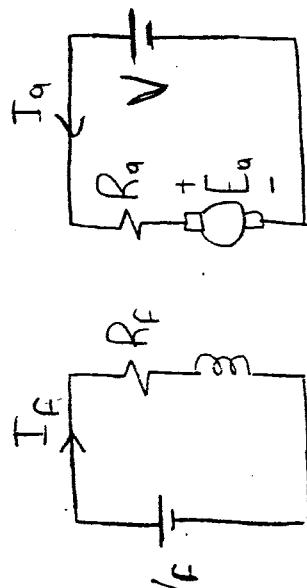
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[XIX] Series**



$$V_F = T_q \frac{1}{T}$$

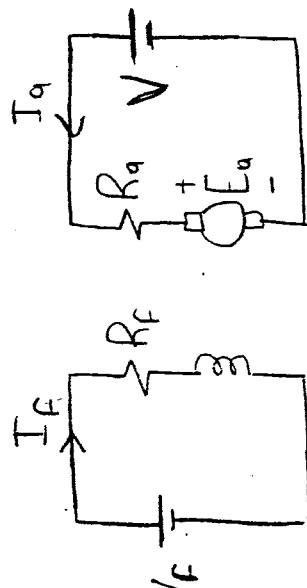
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[XX] Series**



$$V_F = T_q \frac{1}{T}$$

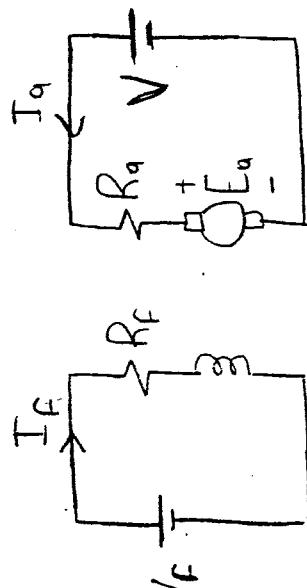
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[XXI] Series**



$$V_F = T_q \frac{1}{T}$$

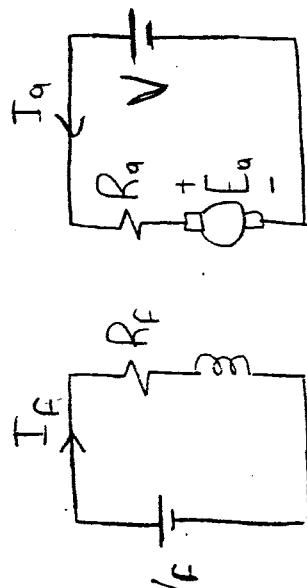
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[XXII] Series**



$$V_F = T_q \frac{1}{T}$$

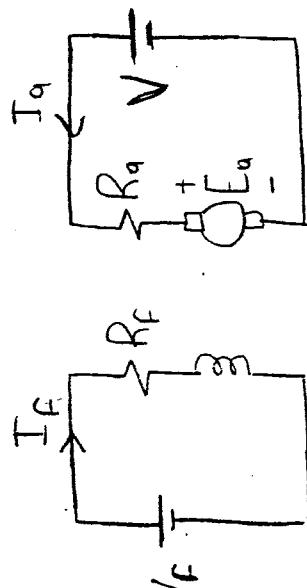
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[XXIII] Series**



$$V_F = T_q \frac{1}{T}$$

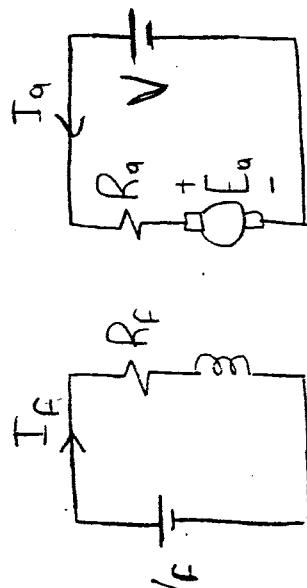
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[XXIV] Series**



$$V_F = T_q \frac{1}{T}$$

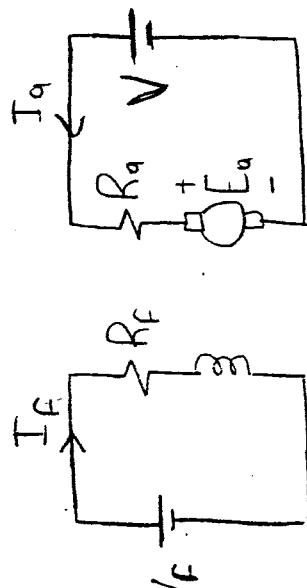
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[XXV] Series**



$$V_F = T_q \frac{1}{T}$$

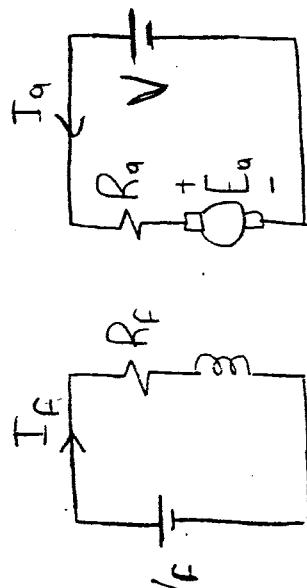
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[XXVI] Series**



$$V_F = T_q \frac{1}{T}$$

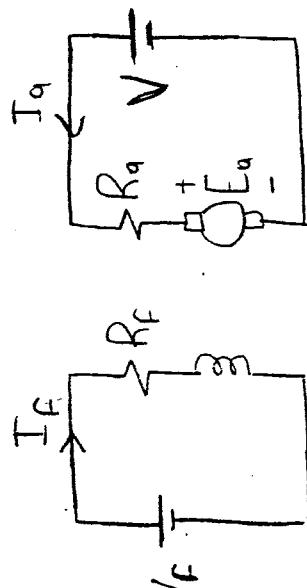
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[XXVII] Series**



$$V_F = T_q \frac{1}{T}$$

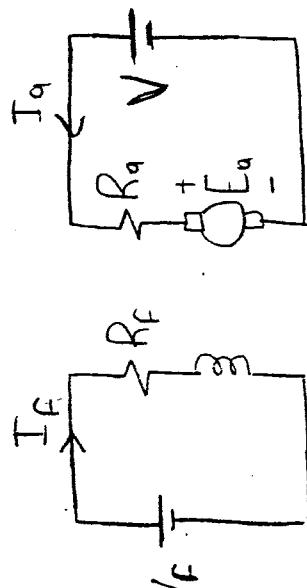
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[XXVIII] Series**



$$V_F = T_q \frac{1}{T}$$

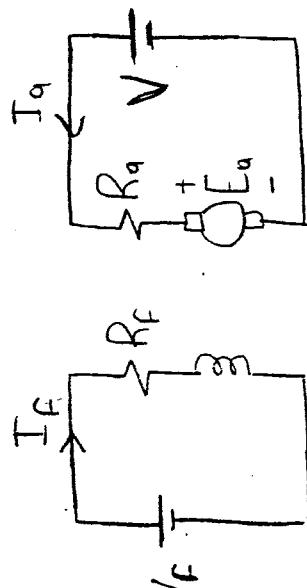
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

**[XXIX] Series**



$$V_F = T_q \frac{1}{T}$$

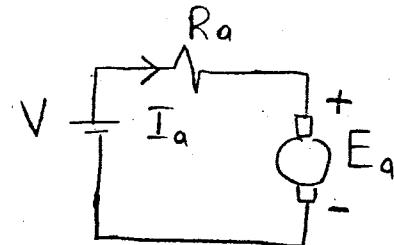
$$E_a = V - T_q R_a$$

$$V_F = T_q R_a$$

$$E_a = V - T_q R_a$$

## Role of back emf ( $E_a$ ) in motors

$E_a$  produces a current opposite to the supply current, so the armature current is limited within acceptable range



## Starting of DC motors

⇒ Answer of Q10

$$\because V = E_a + I_a R_a$$

$$\therefore I_a = \frac{V - E_a}{R_a} \quad \text{but } E_a = K n \Phi$$

But at starting ( $n=0$ )  $\Rightarrow E_a=0$

$$\therefore I_a|_{\text{St.}} = \frac{V-0}{R_a} \uparrow\uparrow \text{ (Very high)}$$

$\therefore$  The armature current is very high at starting, so we must use starters (starting resistors to reduce  $I_a|_{\text{St.}}$ )

## II. problems

(21)

### Laws

$$\textcircled{1} \quad \omega = \frac{2\pi n}{60} \xrightarrow{\text{rad/s}} \text{rpm}$$

$$\textcircled{2} \quad E = K \varphi \omega \rightarrow \textcircled{a}$$

But  $\varphi \propto I_F$ ,  $\omega \propto n$

$$E = K_1 \varphi \cdot n \rightarrow \textcircled{b}$$

$$E = K_2 I_F n \rightarrow \textcircled{c}$$

$$\textcircled{3} \quad T = K \varphi I_a \rightarrow \textcircled{a}$$

$$T = K' I_F I_a \rightarrow \textcircled{b}$$

$$T_d = \frac{P_d}{\omega} \rightarrow \textcircled{c} \quad (T_d: \text{developed torque})$$

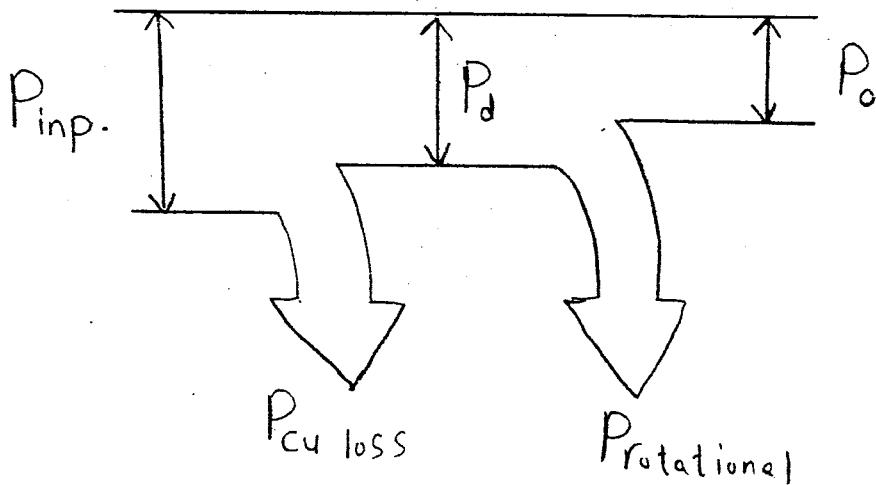
$$\textcircled{4} \quad T_o = \frac{P_o}{\omega} \quad (T_o: \text{output or shaft torque})$$

$$P_o = P_d - P_{\text{friction}}$$

$\textcircled{5}$  IF  $E > V_t \rightarrow$  generator

IF  $E < V_t \rightarrow$  motor

## POWER FLOW IN DC motor



- \*  $P_{\text{inp.}} \equiv \text{input power} = V_s I \xrightarrow{\text{input current}}$
- \*  $P_{\text{Cu loss}} = I^2 R$  (depends on connection)
- \*  $P_d \equiv \text{developed power} = E_a I_a = T_d w_m$
- \*  $P_r \equiv \text{rotational loss}$
- \*  $P_o \equiv \text{output power} = T_{\text{sh}} \downarrow w_m$   
shaft torque
- $$\eta = \frac{P_o}{P_{\text{inp.}}} = \frac{P_{\text{inp.}} - \sum \text{losses}}{P_{\text{in}}}$$
- \*  $P_o = P_d - P_{\text{rotational}}$       \*  $P_d = P_{\text{in}} - P_{\text{Cu loss}}$

## (14) Series motor

sheet

$$R_a = 0.5 \Omega, R_f = 1.5 \Omega$$

$$I_a = 20 \text{ A} \quad \text{When } n = 1200 \text{ rpm}$$

$$V = 220 \text{ V}, P_{\text{rotational}} = 150 \text{ W}$$

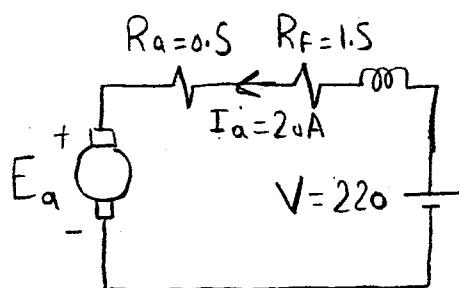
Find  $P_o, \eta$ 

Solution

$$E_a = V - I_a(R_a + R_f)$$

$$E_a = 220 - 20(0.5 + 1.5)$$

$$\therefore E_a = 180 \text{ V}$$



$$\Rightarrow P_o = P_d - P_{\text{rot.}} ; P_d = E_a I_a = 3600 \text{ W}$$

$$\therefore P_o = 3600 - 150$$

$$P_o = 3450 \text{ W}$$

$$\Rightarrow P_i = V * I_a = 4400$$

$$\Rightarrow \eta = \frac{P_o}{P_i} = \frac{3450}{4400} = 78.4\%$$

$$\eta = 78.4\%$$

(IS) • DC motor (Assume separately excited) (24)  
 Sheet

- at No load:  $V = 100V$ ,  $n = 1200 \text{ rpm}$
- $R_a = 2\Omega$
- Find  $T, I_a$  if  $V = 220V$ ,  $n = 1500 \text{ rpm}$
- If is Constant

### Solution

Case ①  $V = 100V$

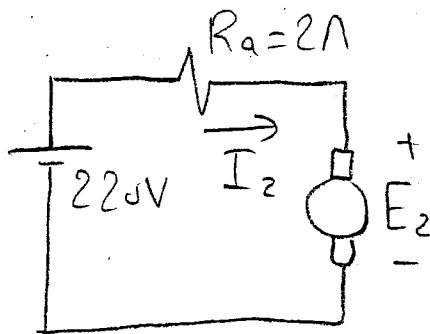
$n_1 = 1200 \text{ rpm}$

No load ( $I_a \approx 0$ )

$$E_1 = 100V$$

case ②  $V = 220V$

$n_2 = 1500 \text{ rpm}$



$$* \frac{E_2}{E_1} = \frac{n_2}{n_1}$$

$$\frac{E_2}{100} = \frac{1500}{1200} \Rightarrow \boxed{E_2 = 125}$$

$$* E_2 = 220 - 2I_2 \Rightarrow \boxed{I_2 = 47.5}$$

$$* P_d = E_2 I_2 = 5937.5W$$

$$* T_d = \frac{P_d}{\omega} = \frac{5937.5}{2\pi \left( \frac{1500}{60} \right)}$$

$$\boxed{T_d = 37.8 \text{ N.m}}$$

16. DC Shunt motor

sheet

- Constant Field ( $I_F = \text{Const.}$ )

- $T \propto n$

- $I_a = 30 \text{ A}$  when  $n_1 = 750 \text{ rpm}$

- $R_{\text{series}} = 10 \Omega \Rightarrow n_2 = ??$

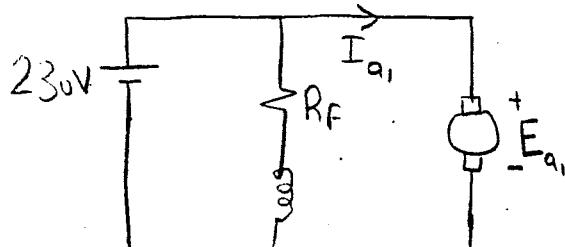
- $R_a$  is neglected

Solution

Case ①

$$n_1 = 750 \text{ rpm}$$

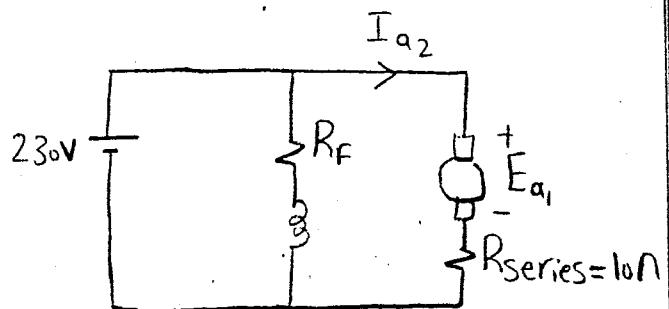
$$I_{a_1} = 30 \text{ A}$$



$$E_{a_1} = 230 \text{ V}$$

Case ②

$$R_{\text{series}} = 10 \Omega$$



$$E_{a_2} = 230 - 10 I_{a_2}$$

$\Rightarrow$  But  $E \propto n \phi^{\text{constant}} \Rightarrow E \propto n$

$$\therefore \frac{E_2}{E_1} = \frac{n_2}{n_1}$$

$$\frac{230 - 10 I_2}{230} = \frac{n_2}{750}$$

$$0.306 N_2 + 10 I_2 = 230 \rightarrow ①$$

$\Rightarrow$  But  $T \propto n$  (given)

$$T \propto \Phi I_a \Rightarrow T \propto I_a$$

const.

$$\therefore \frac{T_2}{T_1} = \frac{n_2}{n_1} = \frac{I_2}{I_1}$$

$$\therefore \frac{n_2}{750} = \frac{I_2}{30}$$

$$\therefore n_2 = 25 I_2 \rightarrow ②$$

Solving ①, ② we get

$$I_{a_2} = 13 A$$

$$n_2 = 325.7 \text{ rpm}$$